## LMT Fall 2024 Guts Round - Part 1

Team Name: _____

_____ 1. **[9]** Find the least prime factor of $2024^{2024} - 1$.

_____ 2. **[9]** A group of nine math team members like to play Survev.io. They noticed that the number of hours each of them played this week forms an arithmetic progression. The person who played the least played for 1 hour, while the most played for 9. Find the total number of hours all nine group members spent playing Survev.io this week.

_____ 3. **[9]** Two distinct positive even integers sum to 8. Find the larger of the two integers.

## LMT Fall 2024 Guts Round - Part 2

Team Name: _____

_____ 4. **[10]** A group of 5 rappers wants to make a song together. They each make their own parts for the song and then arrange the 5 parts. J Cole wants to be friends with both Drake and Kendrick, so he wants his part to be adjacent to both of theirs. Find the number of possible songs (distinct orders) that can be made.

_____ 5. **[10]** Ben Y's favorite number $p$ is prime, and his second favorite number is some integer $n$. Given that $p$ divides $n$ and $n$ divides $3p+91$, find the maximum possible value of $n$.

_____ 6. **[10]** Let $P$ be a point in rectangle $ABCD$ such that the area of $PAB$ is 20 and the area of $PCD$ is 24. Find the area of $ABCD$.

## LMT Fall 2024 Guts Round - Part 3

Team Name: _____

_____ 7. **[11]** Let $L$ be the answer to problem 9. Find the solution to the equation $4x + \sqrt{L} = 0$.

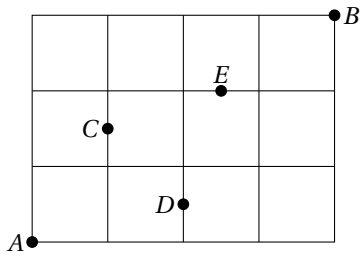
_____ 8. **[11]** Let $M$ be the answer to problem 7. Let $f(x) = x^4 + 4x^3 + 6x^2 + 1$. Find $f(M)$.

_____ 9. **[11]** Let $T$ be the answer to problem 8. Find the area of a square with side length $T$.

## LMT Fall 2024 Guts Round - Part 4

Team Name: _____

_____ 10. **[12]** David starts at the point $A$ and goes up and right along the grid lines to point $B$. At each of the points $C$, $D$, and $E$ there is a bully. Find the number of paths David can take which make him encounter exactly one bully.




_____ 11. **[12]** A Pokemon fan walks into a store. An employee tells them that there are 2 Pikachus, 3 Eevees, 4 Snorlaxes, and 5 Bulbasaurs remaining inside the gacha machine. Given that this fan cannot see what is inside the Poké Balls before opening them, find the least number of Poké Balls they must buy in order to be sure to get one Pikachu and one Snorlax.

_____ 12. **[12]** Snorlax's weight is modeled by the function $w(t) = t2^t$ where $w(t)$ is Snorlax's weight at time $t$ minutes. Find the smallest integer time $t$ such that Snorlax's weight is greater than 10000.

## LMT Fall 2024 Guts Round - Part 5

Team Name: _____

_____ 13. **[13]** Suppose $j$, $x$, and $u$ are positive real numbers such that $jxu = 20$ and $x + u = 24$. Find the minimum possible value of $j\max(x, u)$.

_____ 14. **[13]** Find the number of trailing 0s in the base 12 expression of 99! (Note: 99 is in base 10).

_____ 15. **[13]** Regular hexagon $ABCDEF$ with side length 2 is inscribed within a sphere of radius 4. Let point $X$ be on the sphere. Find the maximum value of the volume of the pyramid $ABCDEFX$.

## LMT Fall 2024 Guts Round - Part 6

Team Name: _____

_____ 16. **[14]** A new meme is circling around social media known as the *DaDerek Convertible*. The license plate number of the *DaDerek Convertible* is such that the product of its nonzero digits times 5 is equal to itself. Given that its license plate number has less than or equal to 3 digits and that it has at least one nonzero digit, find the *DaDerek Convertible*'s license plate number.

_____ 17. **[14]** Suppose $x$, $y$, $z$ are pairwise distinct real numbers satisfying

$$x^2 + 3y = y^2 + 3z = z^2 + 3x.$$

Find $(x + y)(y + z)(z + x)$.

_____ 18. **[14]** In the electoral college, each of 51 places get some positive number of electoral votes for a nationwide total of 538. Thus, 270 electoral votes guarantees a win. Across all distributions of electoral votes to each place, let $M$ be the maximum number of sets of places that combine to have at least 270 electoral votes. Find $M$.

## LMT Fall 2024 Guts Round - Part 7

Team Name: _____

_____ 19. **[15]** Given $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, find

$$\sum_{j=1}^{\infty} \sum_{i=1}^{j} \frac{1}{ij(i+1)(j+1)}.$$

_____ 20. **[15]** A base 9 number *probably places* if it has a 7 as one of its digits. Find the number of base 9 numbers less than or equal to 100 in base 10 that probably place.

_____ 21. **[15]** Let $ABC$ be a triangle such that $AB = 2$, $BC = 3$, and $AC = 4$. A circle passing through $A$ intersects $AB$ at $D$, $AC$ at $E$, and $BC$ at $M$ and $N$ such that $BM = MN = NC$. Find $DE$.

## LMT Fall 2024 Guts Round - Part 8

Team Name: _____

_____ 22. **[17]** Find the number of real numbers $0 \le \alpha < 50$ such that $\alpha^2 + 2\{\alpha\}$ is an integer. (Here $\{\alpha\}$ denotes the fractional part of $\alpha$.)

_____ 23. **[17]** Define $\overline{a}$ of a positive integer $a$ to be the number $a$ with its digits reversed. For example, $\overline{31564} = 46513$. Find the sum of all positive integers $n \le 100$ such that $(\overline{n})^2 = \overline{n^2}$. (Note: For a number that ends with a zero, like 450, the reverse would exclude the zero, so $\overline{450} = 54$).

_____ 24. **[17]** Let $ABC$ be a triangle with $AB = 13, BC = 15, AC = 14$. Let $P$ be the point such that $AP = CP = \frac{1}{2}BP$. Find $AP^2$.

## LMT Fall 2024 Guts Round - Part 9

Team Name: _____

_____ 25. **[20]** Define $f(n)$ to be the sum of positive integers $k$ less than or equal to $n$ such that $\gcd(n,k)$ is prime. Find $f(2024)$.

_____ 26. **[20]** Let $P$ be a point in the interior of square $ABCD$ such that $\angle APB + \angle CPD = 180°$ and $\angle APB < \angle CPD$. If $PC = 7$ and $PD = 5$, find $\frac{PA}{PB}$.

_____ 27. **[20]** Find all positive integer pairs $(a,b)$ that satisfy the equation

$$a^2b + ab^2 + 73 = 8ab + 9a + 9b.$$

## LMT Fall 2024 Guts Round - Part 10

Team Name: _____

_____ 28. **[23]** Find the number of ways to tile a $2 \times 2 \times 2 \times 2$ four dimensional hypercube with $2 \times 1 \times 1 \times 1$ blocks, with reflections and rotations of the large hypercube distinct.

_____ 29. **[23]** Let $P(x)$ be a quartic polynomial with integer coefficients and leading coefficient 1 such that $P(\sqrt{2}+\sqrt{3}+\sqrt{6}) = 0$. Find $P(1)$.

_____ 30. **[23]** Find

$$\sum_{n=1}^{\infty} \frac{\varphi(n)}{(-4)^n - 1},$$

where $\varphi(n)$ is the number of positive integers $k \leq n$ relatively prime to $n$. (Note $\varphi(1) = 1$.)

## LMT Fall 2024 Guts Round - Part 11

Team Name: _____

_____ 31. **[26]** Let $ABC$ be a triangle with circumradius 12, and denote the orthocenter and circumcenter as $H$ and $O$ respectively. Define $H_A \neq A$ to be the intersection of line $AH$ and the circumcircle of $ABC$. Given that $\overline{OH} \parallel \overline{BC}$ and $\overline{AO} \parallel \overline{BH_A}$, find $AH_A$.

_____ 32. **[26]** Let $a$ and $b$ be positive integers such that

$$a^2 + (a+1)^2 = b^4.$$

Find the least possible value of $a+b$.

_____ 33. **[26]** Let $a$ and $b$ be positive real numbers that satisfy

$$\sqrt{a-ab} + \sqrt{b-ab} = \frac{\sqrt{6}+\sqrt{2}}{4} \text{ and } \sqrt{a-a^2} + \sqrt{b-b^2} = \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2.$$

Find the ordered pair $(a,b)$ such that $a > b$ and $a+b$ is maximal.

**LMT Fall 2024 Guts Round - Part 12** Team Name: _____

_____ 34. **[30]** Let a sequence $a_n$ be defined by $a_0 = 0$ and $a_n = na_{n-1} + 2^n$. Estimate $\frac{a_{2024}}{2024!}$.

Submit a positive real number $E$ in decimal form. If the correct answer is $A$, you will receive $\min\left(30, \frac{5}{|E-A|}\right)$ points.

_____ 35. **[30]** Estimate the number of ways to tile a 5 dimensional $2\times2\times2\times2\times2$ *hypercube* with $2\times1\times1\times1\times1$ blocks, with reflections and rotations of the large cube distinct.

Submit a positive real number $E$ in decimal form. If the correct answer is $A$, your score will be

$$\max\left(0, \left\lfloor 30\left(1 - \left|\ln\frac{E}{A}\right|\right)\right\rfloor\right).$$

_____ 36. **[30]** Stan, Kyle, Cartman, and Kenny stand at the four corners of South Park, a square with side length 1 mile. Each of them walks at a random angle between their two adjacent edges at 1 mile per hour. After 1 hour, Kenny spontaneously explodes and dies, killing everyone strictly within a half-mile of him. Estimate the expected number of people who will be killed in this explosion.

Submit a positive real number $E$ in decimal form. If the correct answer is $A$, you will receive $\min\left(30, \left\lfloor \frac{1}{|E-A|^{0.6}} \right\rfloor\right)$ points.